



TECHNICAL BULLETIN — 007 The Per Unit System Overview

Overview

The per unit system is based on the formula shown in Equation (1).

$$(1) \quad \textit{per unit} = \frac{\textit{actual quantity}}{\textit{base quantity}}$$

Using this method, all quantities are expressed as ratios of some base value or values. In many respects the method is similar to the use of percentage values. When analyzing a real system, all parameters such as voltage, current, power, and impedance can be converted to per unit. If the bases are chosen wisely, the entire system may then be analyzed and the local results calculated.

In the following discussion the subscript *pu* will indicate a per unit value, the subscript *B* will indicate a base value, and no subscript will indicate an actual value such as Amperes, Ohms, or Volts.

Fundamental Procedure

The per unit method can be described in five (5) basic steps:

1. Select the base(s) for your system.
2. Convert each known quantity (volts, amps, ohms, etc.) to per unit values.
3. Draw the system impedance diagram using the per unit values.
4. Perform the circuit analysis to determine the current magnitudes in per unit.
5. Convert the per unit currents and voltages from step 4 into Volts and Ohms.

Selection of Base Quantities

The most fundamental requirement of a per unit system is that all formulas and calculations must provide correct answers. In other words, if it works in actual values, it must work in per unit.

Consequently the engineer must carefully select the base values so that they will satisfy the fundamental laws of electricity, including Ohm's law:

$$(2) \quad V_B = I_B Z_B$$

If Equation (2) is true, then Equation (3) is also true:

$$(3) \quad \frac{V}{V_B} = \frac{IZ}{I_B Z_B} \Rightarrow V_{pu} = I_{pu} Z_{pu}$$

To satisfy this need, the base quantities should be selected using the rules given in Table 1.

After the base values have been selected or calculated, then the per unit values in the system can be calculated using Equations (4), (5), and (6).

$$(4) \quad Z_{pu} = \frac{Z \text{ (Ohms)}}{Z_B}$$

$$(5) \quad I_{pu} = \frac{I \text{ (Amperes)}}{I_B}$$

$$(6) \quad V_{pu} = \frac{V \text{ (Volts)}}{V_B}$$

Table 1 Base selection criteria

Base Quantity	Symbol	How Selected
kVA	kVA_B	Select at random. 10,000 kVA or 100,000 kVA are commonly used; however, some equipment (e.g. transformers and generators) have their impedance expressed using their nominal full load as the base.
Volts	kV_B	Select the voltage at the location where equipment is located. For example, a cable located in the 34.5 kV system would use 34.5 kV as its voltage base. See the following discussion for more detail. Note that the line-to-line kV is used.
Current	I_B	$I_B = \frac{kVA_B}{kV_{LL} \times \sqrt{3}}$
Impedance	Z_B	$Z_B = \frac{kV_B \times 1000}{I_B \times \sqrt{3}} \text{ or } Z_B = \frac{kV_B^2 \times 1000}{kVA_B}$

Changing Bases

Occasionally the need arises to convert a per unit value to another base. For example, a transformer nameplate per cent impedance is normally given using the transformer full load kVA as the base. Thus the transformer Z must be converted to the kVA_B being used in the study.

Equation (7) is the requirement for the conversion.

$$(7) \quad Z = Z_{pu}^{old} Z_B^{old} = Z_{pu}^{new} Z_B^{new}$$

This can be re-arranged as:

$$(8) \quad Z_{pu}^{new} = Z_{pu}^{old} \frac{Z_B^{old}}{Z_B^{new}}$$

Substituting:

$$(9) \quad Z_{pu}^{new} = Z_{pu}^{old} \left(\frac{\left((kV_B^{old})^2 \times 1000 \right)}{kVA_B^{old}} \right) \left(\frac{kVA_B^{new}}{\left((kV_B^{new})^2 \times 1000 \right)} \right)$$

Which reduces to:

$$(10) \quad Z_{pu}^{new} = Z_{pu}^{old} \left(\frac{kV_B^{old}}{kV_B^{new}} \right)^2 \frac{kVA_B^{new}}{kVA_B^{old}}$$

Note that the voltage terms in Equation (10) are used only when a device is being employed in a voltage different from the base upon which it was calculated.

Other Useful Formulas

From the previous discussions we know that $kV_B = kV_{LL}$. On a single phase basis then:

$$(11) \quad \frac{V_{LL}}{\sqrt{3}} = I_B Z_B$$

and

$$(12) \quad \frac{V_{LL}}{\sqrt{3}} = I_{sc} Z_{sc}$$

Where the sc subscript indicates *short circuit*. If equation 11 is divided by equation 12 and terms are re-arranged, we have:

$$(13) \quad \frac{I_B}{I_{sc}} = \frac{Z_{sc}}{Z_B}$$

Since:

$$(14) \quad \frac{Z_{sc}}{Z_B} = Z_{pu}$$

Then:

$$(15) \quad Z_{pu} = \frac{I_B}{I_{sc}} \text{ and } Z_{pu} = \frac{kVA_B}{kVA_{sc}}$$

Equation (16) can be developed from Figure 1

$$(16) \quad I_{sc} = \frac{kVA_B}{\sqrt{3} \times kV \times Z_{pu}}$$

Two examples on the following pages will illustrate the use of the per unit system.

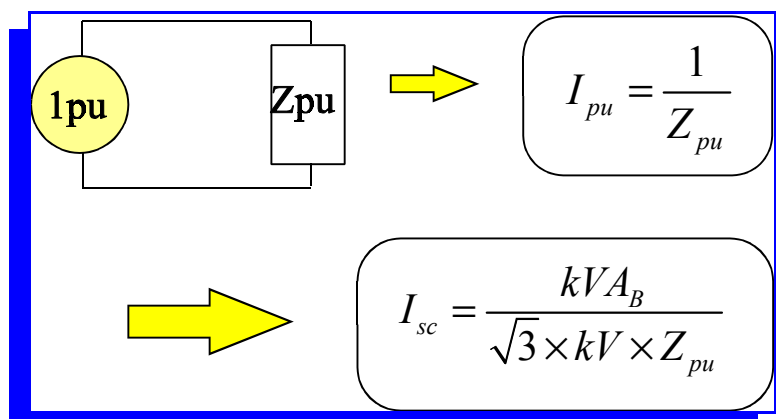


Figure 1 – Development of Equation (16)

Example #1. How is the maximum fault current (I_{sc}) on the secondary side of the transformer in Figure 2 calculated.

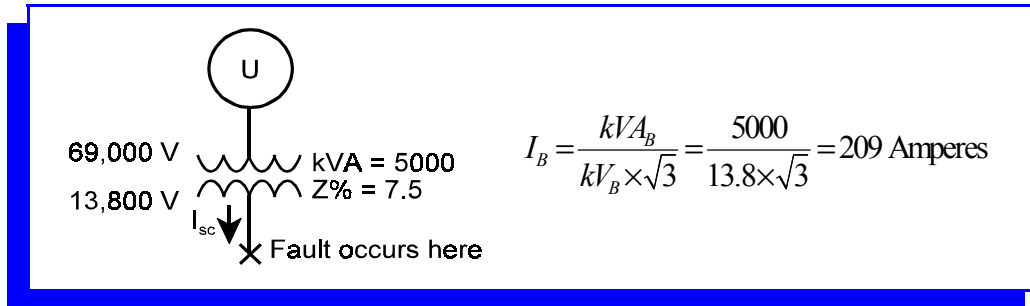


Figure 2 – Fault analysis assuming infinite utility source

Solution:

First we note that:

$$(17) \quad Z_{pu} = \frac{Z\%}{100} = \frac{7.5}{100} = .075 \text{ pu}$$

If we assume that the utility is an infinite source (e.g. has zero impedance) then the maximum current will flow on the transformer secondary. Rewriting Equation 15 slightly gives

$$(18) \quad I_{sc} = \frac{I_B}{Z_{pu}} = \frac{209}{.075} = 2787 \text{ Amperes}$$

Example #2. How is the maximum fault current (I_{sc}) on the secondary side of the transformer in Figure 3 calculated.

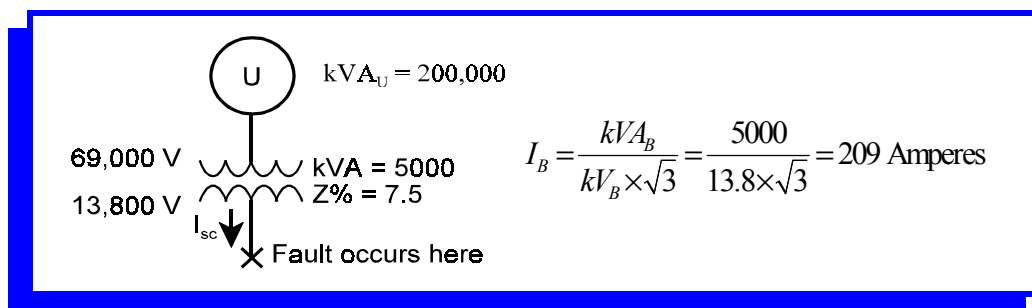


Figure 3 Fault analysis assuming a non-infinite utility

Solution:

In this example, the utility has a maximum short circuit kVA of 200,000. This means that the utility has an impedance which adds to the transformer impedance to further limit the fault current. If we start by assuming a kVA_B of 200,000 for the utility then Equation 15 gives:

$$(19) \quad Z_{pu}(\text{Utility}) = \frac{kVA_B}{kVA_{sc}} = \frac{200,000}{200,000} = 1 pu$$

Next we need to convert the utility impedance to the transformer base using Equation 10

$$Z_{pu}^{new} = Z_{pu}^{old} \left(\frac{kV_B^{old}}{kV_B^{new}} \right)^2 \frac{kVA_B^{new}}{kVA_B^{old}} = 1 \times \frac{69}{69} \times \frac{5000}{200,000} = .025 pu$$

Note that the transformer base voltage is 69 kV since its per unit impedance is the same referenced to either side. The total impedance limiting the short circuit is now the sum of the transformer plus the utility. (We are ignoring phase angle differences in the interest of simplicity.)

Therefore, using Equation 16

$$(21) \quad I_{sc} = \frac{kVA_B}{\sqrt{3} \times kV \times Z_{pu}} = \frac{5000}{\sqrt{3} \times 13.8 \times (.075 + .025)} = 2092 \text{ Amperes}$$